I-5. Application of Exact Synthesis Methods to Multichannel Filter Design

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The major problem in the design of multichannel filters is that of obtaining a network that separates a given frequency band into n-channels with minimum insertion loss and a low VSWR at the input port. The complexity of the structure is determined by the allowable width of the crossover regions between frequency bands and the required isolation between bands.

One general approach is to use a series of two-channel filters (diplexers) as basic building blocks in obtaining the n-channel filter. The general configuration is shown in Fig. 1. The problem then becomes one of obtaining a three-port network with the desired band separation and a low input VSWR over a broad frequency range. Exact synthesis procedures are ideally suited to this type of problem. A general discussion of exact synthesis techniques and their application to microwave networks can be found in the literature.[1, 2, 3, 4]

The basic network to be considered is that of two parallel connected, lossless filters, one lowpass and the other highpass. Each filter is resistively terminated. The desired characteristics are obtained if the filters are designed to be complementary.[5] This requires the input admittances of the low and high-pass filters to obey the relationship,

$$Y_{\text{in}_{LP}} + Y_{\text{in}_{HP}} = 1. \tag{1}$$

Consequently, the sum of the real parts of the input admittances must be a constant and the reactive parts must be equal and opposite. The input VSWR is theoretically unity at all frequencies. For lossless networks terminated in a unit resistance,

$$\text{Re}\,|Y_{\text{in}}| = |Y_{12}|^2, \tag{2}$$

where Y_{12} is the ratio of output current to input voltage. Designing the real parts of Y_{in} to add to a constant is equivalent to requiring that

$$|Y_{12}|^2_{LP} + |Y_{12}|^2_{HP} = 1. \tag{3}$$

The n-th order Butterworth approximation to the low-pass filter characteristic is given by

$$|Y_{12}|^2_{LP} = \frac{1}{1+\Omega^{2n}},$$

and that of its complement by,

$$|Y_{12}|^2_{HP} = \frac{\Omega^{2n}}{1+\Omega^{2n}}. \tag{4}$$

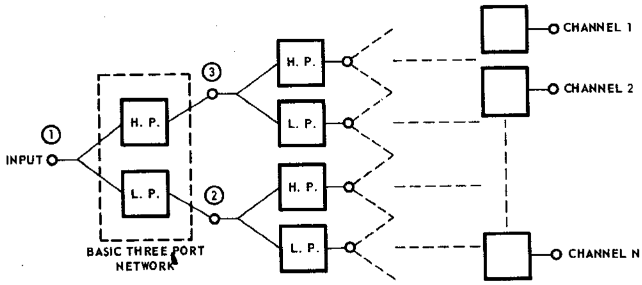

Fig. 1 General configuration of n-channel filter.

The element values of the n-th order filter can be obtained from tables, and a simple method of obtaining the elements of the complement is given in Ref. 4.

With the use of exact synthesis techniques,[4] two third-order wideband Butterworth complements were constructed. A photograph of the filter structure is shown in Fig. 2. The filter transfer characteristics and input VSWR in the frequency range 0–8 Gc are shown in Fig. 3. Sharper crossover characteristics can be obtained by using a higher order filter. If only one crossover region is of interest (as is often the case) maximum VSWR's of less than 1.20 are readily obtainable.

Steeper filter skirts can be obtained for a given number of sections if a Tchebycheff or equal-ripple approximation to the desired characteristic is used. From the viewpoint of input VSWR, the optimum condition occurs when the filters are complementary. However, imposing this condition on Tchebycheff filter pairs leads to poor isolation. This effect can be under-

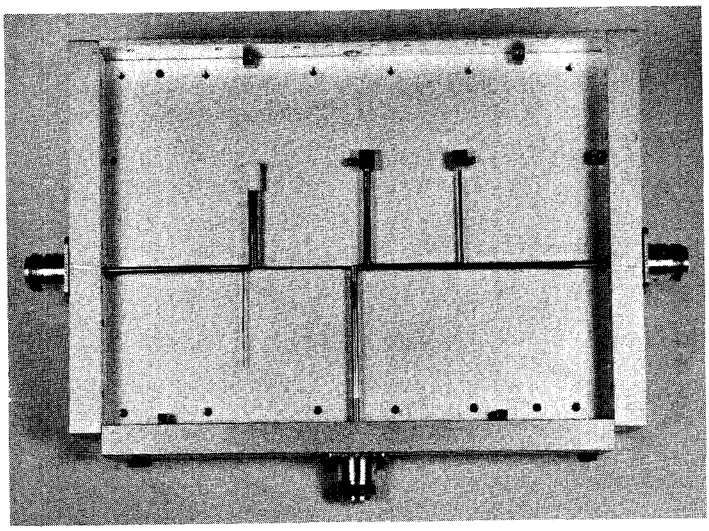

Fig. 2 Third-order Butterworth complementary filters.

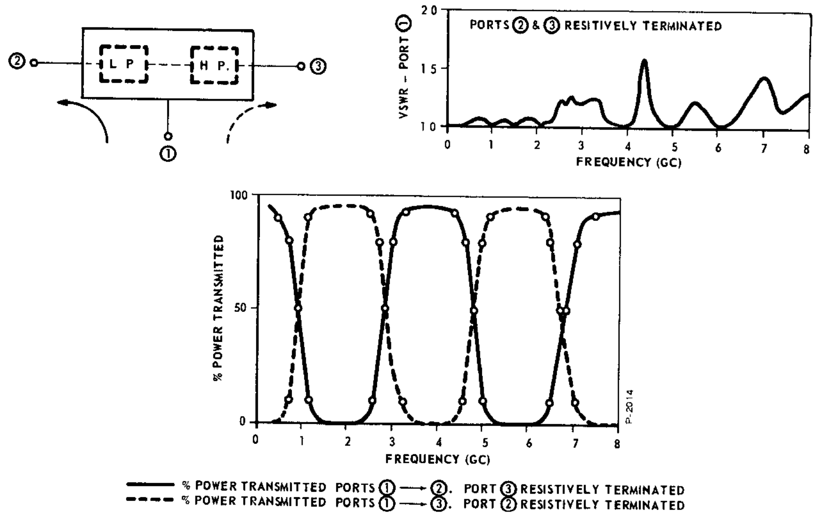

Fig. 3 Measured insertion loss and VSWR characteristics of third-order Butterworth complementary filters.

stood by considering a 0.1 db ripple filter. When the power response of this filter is down 0.1 db, or approximately 2 per cent, the response of a true complement must pass the 2 per cent power, which results in only 17 db isolation. In many practical situations, this amount of isolation is insufficient. Intuitively, one expects that it is possible to improve the isolation characteristic by relaxing the input VSWR requirement.

Insight into the manner in which the VSWR requirement might be relaxed can be obtained by considering the mathematical form of the pertinent filter characteristics. A significant fact is that the input impedances associated with complementary filters are minimum reactive,[6] i.e., they contain no j-axis poles. Under this condition, the impedances are analytic in a half plane and have real and imaginary parts that are Hilbert transforms. Thus,

$$Im\left[Y_{in}(\omega)\right] = -\frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{\mathrm{Re}\left[Y_{in}(x)\right] dx}{\omega - x}. \tag{5}$$

From Equation (2),

$$Im\left[Y_{in}(\omega)\right] = -\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\left[\left|Y_{12}(x)\right|_{LP}^{2} + \left|Y_{12}(x)\right|_{HP}^{2}\right] dx}{\omega - x}. \tag{6}$$

If the networks are complementary Eq. (3) is valid and

$$Im\left[Y_{in}(\omega)\right] = -\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{1}{\omega - x}\, dx = 0. \tag{7}$$

For the functions under consideration, one can integrate or differentiate both functions a number of times, carry out the remaining integrations or differentiations, and then integrate or differentiate so as to undo the initial operation. For example,

$$\frac{d^n Im\left[Y_{in}(\omega)\right]}{d\omega^n} = -\frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{\dfrac{d^n \mathrm{Re}\left[Y_{in}(x)\right]}{dx^n}}{\omega - x}\, dx$$

$$= -\frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{\dfrac{d^n\left[\left|Y_{12}(x)\right|^2_{LP} + \left|Y_{12}(x)\right|^2_{HP}\right]}{dx^n}}{\omega - x}\, dx. \tag{8}$$

The above relations provide a convenient method of calculating (both numerically and graphically) the input VSWR of a filter pair and show what factors most influence the obtainable characteristics. Using criteria derived from these relationships, it is possible to build filter pairs with low input VSWR and crossover characteristics that are significantly better than the Butterworth characteristic of comparable order. These filters are synthesized with the use of an exact theory and thus have characteristics that can be specified over extremely wide frequency bands.

As a simple example of the application of the above relations, consider the piecewise linear lowpass-highpass filter pair characteristics shown in Fig. 4. First and second derivatives of the characteristics are shown, respectively, in Fig. 4 (b) and 4 (c). Since the functions have been reduced to

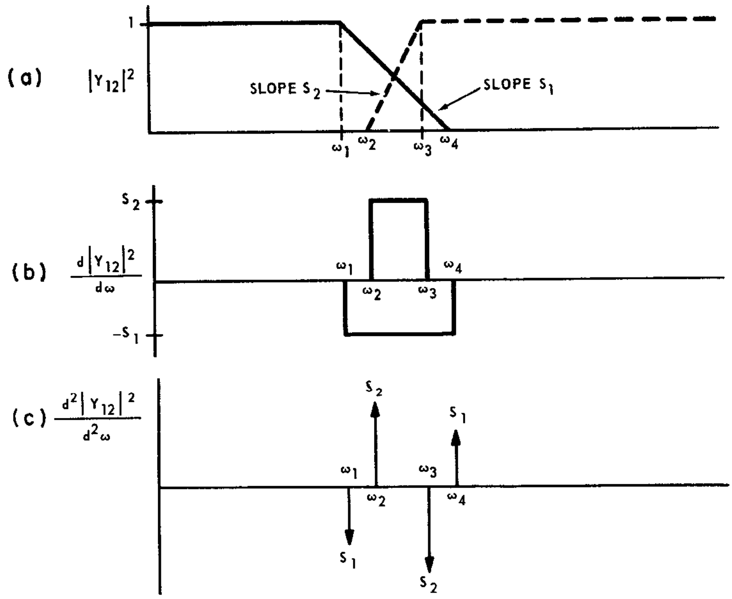

Fig. 4 (a) Piecewise linear lowpass-highpass filter characteristics $\left[|Y_{12}|^2 = \mathrm{Re}\,(Y_{in})\right]$. (b) First derivative of filter characteristics. (c) Second derivative of filter characteristics.

a series of impulses, the integral (Eq. (8)) can be evaluated by inspection and the imaginary part can be determined by two integrations of the resulting function. A good design criterion can be obtained from Fig. 4 (c). Location of the impulses is determined by the filter break points, and their magnitudes are determined by the slope of the filter characteristics. Thus, if the break points are forced to occur at the same frequencies and the slopes of the filter characteristics are made equal and opposite, the impulses will cancel, resulting in a zero imaginary part. Under these conditions, the real parts must add to unity (Eq. (7)), as in the case of true complements. For the case of Tchebycheff filter characteristics, if high isolation is desired, the filters cannot be made truly complementary, and therefore the imaginary part will be non-zero. An analysis similar to that of the example above can be used to obtain suitable design criteria. The resulting networks are realized by the use of exact synthesis methods.

REFERENCES

1. E. A. Guillemin, *Synthesis of Passive Networks*, (New York: Wiley, 1957).
2. L. Weinberg, *Network Analysis and Synthesis*, (New York: McGraw-Hill, 1962).
3. H. Ozaki and J. Ishii, "Synthesis of a Class of Strip-Line Filters," *IRE Trans. on Circuit Theory*, Vol. CT-5, pp. 108-109 (June 1958).
4. R. J. Wenzel, "Exact Design of TEM Microwave Networks Using Quarter-Wave Lines," *IEEE Trans. on Microwave Theory and Techniques*, January 1964.
5. A thorough discussion of complementary filters can be found in Ref. 1 and examples of microwave complements in Ref. 4.
6. The input impedances (admittances) may be minimum reactive or minimum susceptive. It is important to keep in mind that the impedances (admittances) do not have to be both. See Guillemin, Ref. 1, p. 474.

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